Comparative Analysis of Market Structures: Theory, History, and Application to Continuous/Distribution Markets

I. Introduction

Context:
The landscape of financial markets is undergoing a period of rapid evolution, driven significantly by technological advancements, particularly the emergence of blockchain technology and the decentralized finance (DeFi) ecosystem. This evolution has spurred the development and deployment of novel market mechanisms that challenge and complement traditional structures. While established paradigms like order-driven limit order books and quote-driven dealer markets are well-documented within the field of market microstructure 1, newer forms such as prediction markets, parimutuel betting systems, and especially Automated Market Makers (AMMs) present unique operational characteristics, economic implications, and design trade-offs.3 The recent theoretical proposal of 'distribution markets'—markets designed for trading contracts based on the entire probability distribution of a continuous outcome—further expands the frontier of market design, aiming to capture richer informational nuances than previously possible.6
Problem Statement:
Despite the proliferation and increasing economic significance of these diverse market structures, a comprehensive, comparative analysis grounded in rigorous academic theory and employing consistent terminology remains relatively scarce. This gap hinders the ability of researchers, practitioners, and market designers to make informed choices, particularly when developing sophisticated applications such as novel blockchain-based distribution markets. Key areas requiring systematic clarification include the fundamental consequences of different asset scarcity models on market dynamics, the precise role and net economic impact of various liquidity provision mechanisms, and the distinct properties, advantages, and disadvantages of specific AMM algorithms like Constant Function Market Makers (CFMMs) versus Logarithmic Market Scoring Rule (LMSR) based systems.5 Without a clear theoretical and historical foundation, designing efficient, robust, and fit-for-purpose markets for complex claims like probability distributions becomes an ad hoc exercise, potentially leading to suboptimal outcomes, inefficiencies, or unforeseen risks.11
Research Objectives:
This report undertakes an expert-level comparative analysis of diverse market structures, aiming to build a foundational understanding relevant to the design of innovative market mechanisms, particularly blockchain-based distribution markets. The analysis synthesizes theoretical frameworks, historical context, and practical implications, adhering to established academic terminology. Specifically, this report addresses the following core research objectives:

1. Analyze the fundamental differences and implications of markets trading two inherently scarce assets versus those where one side represents contracts creatable on demand (Asset Scarcity) [Obj 1].

2. Evaluate the theoretical role and net economic impact of external liquidity providers (LPs) compared to endogenous liquidity mechanisms (Liquidity Provision) [Obj 2].
3. Provide a rigorous definition and comparative analysis of the parimutuel market structure (Parimutuel Markets) [Obj 3].
4. Clarify the theoretical and terminological distinctions between continuous outcome markets and distribution markets, analyzing their respective mechanisms (Continuous vs. Distribution Markets) [Obj 4].
5. Conduct a detailed comparative analysis of Constant Product CFMMs and LMSR-based AMMs, focusing on their properties and internal liquidity mechanisms (AMM Comparison) [Obj 5].
6. Perform a historical review of academic literature and implementations related to distribution market concepts and complex contingent claims (Historical Review) [Obj 6].

Methodology:
The analysis presented herein integrates theoretical frameworks from market microstructure 1, asset pricing theory 2, information economics, game theory, and mechanism design. Where appropriate, mathematical formalisms and comparisons are employed to clarify the properties of specific mechanisms, such as AMM trading functions.5 The historical review draws upon seminal academic papers 17 and notable real-world implementations across various market types. Throughout the report, emphasis is placed on the precise definition and consistent application of established academic and industry terminology.
Report Structure:
Following this introduction, Section II establishes foundational concepts from market microstructure theory, providing the necessary vocabulary and theoretical grounding. Section III delves into the critical implications of different asset scarcity models. Section IV analyzes the role and net impact of external versus endogenous liquidity provision. Section V provides a detailed examination of the parimutuel market structure. Section VI clarifies the distinction between continuous outcome markets and the newer concept of distribution markets. Section VII offers a comparative analysis of CFMM and LMSR automated market makers. Section VIII presents a historical review of concepts relevant to distribution markets and the trading of complex contingent claims. Finally, Section IX synthesizes the findings across all objectives, explicitly drawing out implications for the design of blockchain-based distribution markets and offering concluding remarks.

II. Foundational Concepts in Market Microstructure

Defining Market Microstructure:
Market microstructure is the subfield of finance dedicated to understanding the processes and outcomes involved in exchanging assets under specific, explicit trading rules.1 While much of traditional economic and financial theory abstracts away from the mechanics of how trades actually occur, market microstructure places these mechanics at the center of its

analysis.1 It investigates how the institutional setting and the specific rules governing trading influence critical market variables such as price formation, transaction costs, market liquidity, trading volume, and the strategic behavior of market participants.1 Maureen O'Hara defines it as “the study of the process and outcomes of exchanging assets under explicit trading rules” 1, while Ananth Madhavan characterizes it as the study of "the process by which investors' latent demands are ultimately translated into prices and volumes".12 Once the simplifying assumptions of frictionless and instantaneous trading are relaxed, the rules governing exchange become paramount.2 The field addresses fundamental questions regarding market structure and design, price formation and discovery, the nature and impact of transaction costs, sources of price volatility, the role of information and its disclosure (transparency), the determinants and measurement of liquidity, and the behavior of diverse market participants.1 Understanding these elements is crucial not only for academics but also for practitioners, regulators, and market designers seeking to evaluate or create efficient and robust trading venues.2

Core Market Mechanisms:

Different markets employ distinct mechanisms to facilitate trade, each with unique properties:

- **Order-Driven Markets (Limit Order Books - LOBs):** These markets operate based on orders submitted directly by participants, without relying on designated intermediaries to quote prices.[23] The central feature is the limit order book, an electronic record displaying outstanding orders to buy (bids) and sell (asks) at specific price levels.[25] Participants submit *limit orders*, which specify a maximum price for buying or a minimum price for selling, thereby providing liquidity to the market and signaling conditional trading interest.[21] Limit orders are contingent on price. Alternatively, participants can submit *market orders*, which instruct the system to execute the trade immediately at the best available price(s) currently in the order book.[21] Market orders demand immediacy and consume liquidity provided by limit orders. Trading occurs when an incoming market order crosses the spread and executes against resting limit orders, or when two limit orders cross.[26] Examples include major stock exchanges like the NYSE (in its electronic form) and the Paris Bourse.[27]
- **Quote-Driven (Dealer) Markets:** In these markets, designated intermediaries, known as dealers or market makers (MMs), play a central role by continuously posting prices at which they are willing to buy (bid price) and sell (ask price) a particular asset.[2] Investors trade *with* these dealers, rather than directly with each other. The primary service provided by dealers is *immediacy*—the ability for investors to execute trades instantly at the quoted prices.[26] Dealers profit from the *bid-ask spread* (the difference between their ask and bid prices) but bear risks associated with holding inventory and trading against potentially better-informed counterparties.[22] Examples include the Nasdaq market maker system [26], many bond markets, and foreign exchange (FX) markets.[26]

- **Auction Markets:** These mechanisms determine prices through formal bidding processes. *Call auctions* aggregate orders over a period and execute them all at a single clearing price at a specific point in time.[25] This mechanism is often used for market openings or for less liquid assets.[25] *Continuous double auctions (CDAs)* allow participants to submit bids and asks continuously, with trades occurring whenever a bid and ask match or cross.[33] CDAs are common in many financial exchanges and are also used in some prediction markets.[33]

Price Discovery and Information Aggregation:
A central function of any market is price discovery, the process through which the market price converges towards the asset's fundamental or equilibrium value by incorporating relevant information.25 It is the mechanism by which the latent demands and supplies of investors, driven by their information and beliefs, are translated into observable transaction prices.12
In LOBs, price discovery is a dynamic process involving both market and limit orders.[23] Market orders reveal immediacy demand and can signal information through their direction and size, impacting prices by consuming liquidity.[23] Limit orders, through their placement, modification, and cancellation, signal traders' valuations and beliefs at specific price points, shaping the visible liquidity landscape and influencing subsequent price movements.[23] Research suggests that high-frequency traders (HFTs) often contribute significantly to price discovery via their strategic placement and rapid adjustment of limit orders.[23]

The presence of *information asymmetry*—where some traders possess private information unknown to others—is a key driver of price discovery, particularly emphasized in information-based microstructure models.[2] Seminal models like Kyle (1985) [2] and Glosten and Milgrom (1985) [2] analyze how informed traders strategically reveal their information through trading, while market makers or uninformed traders attempt to infer this information from the order flow. This process causes prices to gradually adjust towards the informed traders' valuation, but also creates *adverse selection* risk for liquidity providers, who may unknowingly trade at unfavorable prices against informed counterparties.[12] Prediction markets are explicitly designed around the principle of information aggregation, aiming to synthesize dispersed pieces of information held by participants into a collective forecast represented by the market price.[40] Their effectiveness hinges on the market's ability to incentivize information revelation and accurately reflect the collective belief.[41]

Liquidity:
Liquidity is a multifaceted concept referring to the ability to trade an asset quickly, in substantial size, and without causing a significant adverse movement in its price.1 It is widely considered a crucial indicator of market quality and efficiency.1 Key dimensions include:

- **Width:** The cost of executing a small market order, typically measured by the bid-ask spread. A narrower spread indicates higher liquidity.[1]
- **Depth:** The volume of orders available at prices near the best bid and ask. A deeper market can absorb larger orders with less price impact.[1]
- **Resilience:** The speed at which prices revert to fundamental values following a large, potentially price-disrupting trade.[2]
- **Immediacy:** The speed at which trades can be executed at the prevailing market price.[2]

Liquidity is influenced by numerous factors, including the market's structure (order-driven vs. quote-driven vs. AMM) [1], specific trading rules like tick size [1], the presence and behavior of liquidity providers [1], the degree of information asymmetry [2], and asset price volatility.[48]

Transaction Costs:
Trading involves costs beyond the nominal price of the asset. Market microstructure theory places significant emphasis on analyzing these costs.1 They can be categorized as:

- **Explicit Costs:** Direct, observable costs such as brokerage commissions, exchange fees, and taxes.[2]
- **Implicit Costs:** Indirect costs embedded in the trading process itself. These include:
  - *Bid-Ask Spread:* The difference between the best ask price and the best bid price, representing the cost of demanding immediacy.[1]
  - *Price Impact:* The adverse price movement caused by the execution of one's own trade, particularly significant for large orders.[2]
  - *Adverse Selection Costs:* The component of the spread that compensates liquidity providers for the risk of trading with better-informed counterparties.[1]
  - *Inventory Holding Costs:* Costs borne by dealers for holding assets in inventory, including financing costs and risk exposure.[1]
  - *Opportunity Costs:* Costs associated with delays or failure in executing a desired trade.

These transaction costs directly affect investor returns, influence trading strategies and execution methods, and impact overall market efficiency and liquidity.[1]

Key Microstructure Models:
Two primary classes of theoretical models dominate the early microstructure literature:

- **Inventory Models:** Pioneered by researchers like Demsetz (1968), Bagehot (1971), and Stoll (1978) [2], these models focus on quote-driven markets. They explain the bid-ask spread as compensation for the costs and risks dealers incur in managing

their inventory of securities.[2] Dealers adjust their quotes strategically based on their current inventory levels (e.g., lowering bids and asks when holding excess inventory) and their perception of inventory risk (driven by factors like asset volatility).[22] These models highlight the market maker's role in providing immediacy by absorbing temporary order imbalances through their inventory.[22]
- **Information-Based Models:** Beginning with work by Copeland and Galai (1983), Glosten and Milgrom (1985), and Kyle (1985) [2], these models emphasize the impact of asymmetric information. They posit that some traders possess private information about an asset's true value, while others trade for liquidity or portfolio reasons (noise traders).[2] Liquidity providers (market makers or limit order traders) face adverse selection risk, as they may unknowingly trade with informed traders at prices that are about to become disadvantageous.[22] The bid-ask spread, in this view, includes a component to compensate for expected losses to informed traders.[12] These models explain how private information becomes gradually incorporated into prices through the trading process.[2]

The foundational concepts and models of market microstructure provide an essential lens for analyzing *any* trading mechanism. The design choices inherent in creating a novel market, such as a blockchain-based distribution market, directly correspond to the core variables studied by microstructure—trading rules, order types, liquidity provision, and information handling.[1] Neglecting these principles risks creating a market that is inefficient, illiquid, or susceptible to manipulation. The inherent trade-offs identified by microstructure theory, such as the balance between guaranteed immediacy (often provided by intermediaries) and the costs associated with that provision (spreads, risks), are directly relevant when considering different approaches to liquidity, such as relying on external providers versus fostering endogenous liquidity.[22] A deep understanding of these fundamentals is therefore not merely academic background but a prerequisite for effective market design.

III. Asset Scarcity: Contrasting Market Models

Defining Scarcity in Economics:
Scarcity is a cornerstone concept in economics, representing the fundamental tension between potentially unlimited wants and needs and the finite availability of resources to satisfy them.49 An item is considered economically scarce if the demand for it would exceed its supply if it were offered at zero cost.50 This fundamental constraint forces economic agents—individuals, firms, and governments—to make choices about resource allocation, inevitably leading to opportunity costs, which represent the value of the next best alternative foregone.49 Scarcity directly influences the monetary value attributed to goods and services; generally, greater scarcity, relative to demand, tends to increase an item's price or value.49 This relationship is governed by the laws of supply and demand, where scarcity (a decrease in

supply relative to demand, or an increase in demand relative to supply) shifts the market equilibrium towards a higher price.49 Scarcity can arise from various sources, including demand surges (demand-induced), supply disruptions or depletion (supply-induced), or institutional factors like mismanagement or inequality (structural scarcity).49 Furthermore, scarcity is often relative; for instance, in production, one input factor (like specialized labor or land) might be relatively scarcer than another, constraining output even if the other factor is abundant.49

Model 1: Markets with Two Inherently Scarce Assets (e.g., Equities/USD):

Traditional financial markets, such as those for equities, bonds, or commodities traded against fiat currencies, typically operate under a model where both assets involved in a transaction are subject to inherent scarcity constraints.

- **Characteristics:** The supply of equities is determined by corporate issuance decisions, which are influenced by factors like firm size, growth prospects, market conditions, and corporate governance preferences (e.g., reluctance of family firms to dilute control).[52] The total outstanding stock is finite at any given time. Similarly, the supply of major fiat currencies like the US Dollar is managed by central banks through monetary policy, aiming to control inflation and maintain economic stability; money is intentionally kept relatively scarce.[49] A significant theme in recent macroeconomic literature is the concept of *safe asset scarcity*, where the demand for low-risk assets like high-quality government bonds outstrips the supply provided by sovereign issuers, driven by factors like global savings gluts, regulatory requirements, and demand for collateral.[52] Even abundant natural resources can become scarce through depletion.[49]
- **Implications of Bilateral Scarcity:**
  - **Price Formation:** Equilibrium prices are determined by the interaction of supply and demand for *both* assets involved in the pair.[51] A change in the scarcity of either asset (e.g., increased demand for safe bonds, reduced supply of equities) directly impacts the relative price.[49] Asset shortages, where demand persistently outstrips supply at prevailing prices, can contribute to asset price bubbles, volatility, and potential financial crises.[52] Central bank interventions like quantitative easing (QE), which involve large-scale purchases of assets like government bonds, directly manipulate scarcity, impacting yields and market functioning.[54]
  - **Liquidity:** Asset scarcity is intrinsically linked to market liquidity. When specific assets become scarce relative to demand, finding counterparties becomes more difficult, search costs increase [57], and market liquidity tends to decline, manifesting as wider bid-ask spreads and reduced market depth, particularly for the scarce asset.[54] This can lead to a bifurcation where liquidity concentrates in less scarce, more actively traded instruments, while deteriorating for others.[48] Asset scarcity can also impair the ability of the

financial system to provide liquidity insurance.[58] Conversely, policies that alleviate scarcity, such as central banks accepting broader collateral, can support market functioning.[55]

  - **Arbitrage:** Arbitrage opportunities arise from deviations of market prices from perceived fundamental values or from price discrepancies across different markets for the same scarce asset. However, executing arbitrage strategies requires obtaining the mispriced assets, which can be difficult or costly if those assets are scarce or illiquid. The ability to borrow scarce assets (e.g., for short selling) is also crucial and can be constrained.
  - **Intermediation:** Financial intermediaries, such as dealers, market makers, and banks, play vital roles in these markets.[22] They bridge the gap between buyers and sellers, manage inventories of scarce assets to provide immediacy, and facilitate price discovery. Their capacity to perform these functions is often limited by their own capital, risk tolerance, and funding constraints.[22] Scarcity of assets, particularly safe assets, heavily influences collateralized lending markets (like repo), affecting funding costs and the transmission of monetary policy.[55]

Model 2: Markets with On-Demand Contract Creation (e.g., Prediction Markets):
A distinct class of markets involves trading where one 'side' represents a contingent contract or synthetic asset whose supply is not directly tied to physical constraints or external issuance limits but is instead determined endogenously by the market mechanism itself.

- **Characteristics:** Examples include prediction markets trading binary options or event derivatives [36], markets for synthetic assets designed to track the price of other assets [59], and potentially distribution markets.[6] The key feature is that the contract representing a claim on a future outcome or state can, in theory, be created by the market mechanism as needed to meet trader demand.
- **Contract Creation Mechanisms:**
  - *Counterparty Matching (e.g., CDA):* In a continuous double auction setting, a contract effectively comes into existence when a buyer's bid meets a seller's ask.[36] The supply of contracts depends on the willingness of participants to take the opposite side of a bet.
  - *Market Maker / AMM as Counterparty:* A central entity, either a traditional market maker or an automated market maker (AMM), acts as the counterparty to all trades.[36] When a trader wants to buy a contract (e.g., betting 'Yes' on an event), the MM/AMM creates and sells that contract, taking collateral (e.g., USD, stablecoins, ETH) in return. The MM/AMM synthesizes the supply of the contract on demand.[59] Prominent examples include LMSR-based prediction market makers [9] and dynamic parimutuel markets (DPMs).[35] CFMMs can also

be used to create synthetic exposures.[59]

- *Parimutuel Issuance:* In many parimutuel systems, participants buy 'tickets' or shares representing a bet on a specific outcome, often at a fixed price (e.g., $1).[4] The supply of these tickets is typically unlimited, meaning the supply curve for entering a bet is perfectly elastic at the initial price.[63]

- **Implications of Elastic Contract Supply:**
  - **Price Formation:** Prices (which often represent probabilities in prediction markets [41]) are determined primarily by the aggregation of trader beliefs and demands as they interact with the specific market mechanism.[36] Because the contract supply can be highly elastic, the price is less influenced by supply constraints of the contract itself and more by the marginal trader's willingness to bet [44] or the internal state and parameters of the AMM.[5] The supply curve for the *contract* is shaped by the mechanism, contrasting with the externally determined supply curves for scarce assets.[51]
  - **Market Dynamics & Liquidity:** These markets can potentially offer very high, even theoretically infinite, liquidity for *entering* positions, especially when an AMM acts as the counterparty, always ready to create a contract at some price.[34] Liquidity is thus a function of the mechanism's design (e.g., the 'b' parameter in LMSR determining price sensitivity [9]) and the capital or collateral available to the market maker/AMM, rather than the physical availability of the traded instrument. However, *overall* market liquidity can still be low if participation (the number of traders or the amount of collateral) is limited, leading to thin markets or high slippage despite the elastic contract supply.[37]
  - **Arbitrage:** Arbitrage opportunities primarily arise from discrepancies between market-implied probabilities and traders' assessments of true probabilities, or from internal inconsistencies within the market (e.g., probabilities across related contracts not summing to one, or deviations from AMM invariants [5]). Arbitrageurs play a crucial role in enforcing consistency and driving prices towards informational efficiency.[5] Executing arbitrage is less constrained by the need to locate or borrow the *contract* itself (as it can be created) and more by the availability of capital and the transaction costs or slippage imposed by the mechanism.
  - **Intermediation:** The role of traditional intermediaries shifts significantly. In AMM- or MM-driven markets, the mechanism *itself* acts as the primary intermediary for contract creation and liquidity provision.[9] It programmatically quotes prices and takes the other side of trades. The need for specialized dealers holding inventories of the *contract* is eliminated, although intermediaries may still be involved in managing the *collateral* asset (e.g., stablecoin issuers, fiat on/off ramps). Parimutuel markets inherently centralize

the pooling and payout function, removing the need for bilateral intermediation for the betting process itself.[4]

The fundamental distinction between these two models lies in the *locus of scarcity*. In traditional markets (Model 1), the scarcity resides in the traded *assets* themselves – the finite number of shares, bonds, or barrels of oil.[49] In markets with on-demand contract creation (Model 2), the scarcity constraint shifts. While the *contract* representing a bet or synthetic exposure can be created elastically by the mechanism, the ultimate limiting factor becomes the *collateral* or *capital* required to back these contracts and ensure the solvency of the market maker or the pool.[37] The mechanism can issue infinite contracts in theory, but only if it has sufficient backing or if participants provide enough collateral to cover potential payouts.

This shift has profound implications. Because the contract supply is endogenous to the mechanism in Model 2, the *design of that mechanism*—the specific AMM formula, the parimutuel rules, the collateral requirements—becomes the primary determinant of market behavior, liquidity characteristics, price dynamics, and overall efficiency.[5] In Model 1, while market rules certainly matter [1], the inherent scarcity of the underlying assets imposes significant external constraints on market outcomes, regardless of the mechanism's design.[49] This underscores the critical importance of mechanism design for the user's goal of creating a novel distribution market, as the chosen mechanism will fundamentally shape the market's properties.

Furthermore, blockchain technology is particularly well-suited for implementing Model 2 markets. Smart contracts can automate the complex logic of AMMs or parimutuel systems, ensuring transparent and deterministic contract creation, collateral management, and settlement without reliance on traditional, centralized intermediaries.[3] This capability greatly facilitates the practical realization of markets where contract supply is elastic and mechanism-driven.

IV. Liquidity Provision: External vs. Endogenous Mechanisms

The Role of Liquidity Providers (LPs) and Market Makers (MMs):
Liquidity providers, in the broadest sense, are entities that facilitate trading by making it easier for others to buy or sell assets without significantly impacting the price. Market makers (MMs) are a specific type of LP, typically professional firms or individuals, who actively quote two-sided prices (bid and ask) at which they stand ready to trade, thereby providing immediacy to the market.21 Their core function is to bridge temporal gaps between the arrival of buyers and sellers, absorbing temporary order imbalances and ensuring continuous trading opportunities.22
The mechanism by which liquidity is provided varies across market structures. In

traditional quote-driven markets, dealers are the primary LPs, actively setting prices.[22] In order-driven markets, liquidity can be provided passively by any participant who submits limit orders, creating the order book against which market orders execute.[23] In Automated Market Maker (AMM) systems common in DeFi, LPs are typically users who deposit pairs of assets into a shared liquidity pool, governed by a smart contract.[5]

Providing liquidity is not without cost or risk. LPs face *inventory risk*—the risk that the value of the assets they hold will change adversely while in their possession.[2] They also face *adverse selection risk*—the risk of trading against counterparties who possess superior information about the asset's future price, leading to systematic losses.[2] Additionally, there are operational or *order processing costs* associated with maintaining quotes and executing trades.[26] In the context of AMMs, LPs face a unique risk known as *impermanent loss* (or divergence loss), which arises when the relative price of the assets in the pool changes compared to the price at the time of deposit.[5] To compensate for these costs and risks, LPs earn revenue, typically through the bid-ask spread in traditional markets [22] or through trading fees charged to users in AMM pools.[16]

External Liquidity Provision (Dedicated MMs/LPs):
This model relies on specialized, often professional, entities whose primary business is to provide liquidity to the market. These include traditional dealers on stock exchanges or in OTC markets, and high-frequency trading (HFT) firms acting as market makers on electronic platforms.22

- **Theoretical Analysis (Net Impact):** The presence of dedicated external LPs generally enhances market quality. Theoretically and empirically, they contribute to *increased market depth* and *tighter bid-ask spreads*, which translates to lower transaction costs for end-users.[72] By constantly quoting prices, they facilitate more efficient *price discovery*, helping prices reflect available information more quickly.[78] Their willingness to absorb temporary order imbalances contributes to *market stability* and allows for the execution of larger trades with less price impact.[48] Studies suggest that introducing dedicated LP schemes improves market quality metrics.[77] The positive impact may be more pronounced during periods of market stress or illiquidity, where external LPs can step in to absorb selling pressure.[54]

  However, external liquidity provision is not a panacea. LPs charge for their services, primarily through the bid-ask spread or explicit fees, representing a cost to traders.[26] If the market making industry is concentrated, LPs might exercise market power, leading to wider spreads than in a perfectly competitive scenario.[46] A significant concern is *liquidity fragility*. External LPs operate under risk limits and capital constraints.[46] During periods of high volatility or market stress, they

may widen their spreads dramatically or withdraw from the market altogether to protect their capital, causing liquidity to evaporate precisely when it is most needed.[45] HFT LPs, while often narrowing spreads in normal times, have also been implicated in increasing short-term volatility and contributing to flash crashes through rapid order cancellations or liquidity withdrawal.[76] Furthermore, potential conflicts of interest can arise, for example, when a broker also acts as a market maker against its own clients.[72]
The net benefit of external LPs depends on the context. They are most valuable in markets where the natural flow of orders from end-users is insufficient to maintain continuous liquidity and tight spreads.[77] Their impact is modulated by market conditions, regulatory frameworks, and the level of competition among LPs.[45]

- **Mathematical Exploration:** Microstructure models analyze the determinants of the spread quoted by external LPs, linking it to inventory costs, adverse selection risk, order processing costs, and market power.[26] Welfare analyses attempt to quantify the trade-off between the reduction in trading costs enjoyed by users due to LP activity and the costs incurred or profits earned by the LPs themselves.[11] Models like Vayanos and Wang (2010) provide a unified framework to study how various market imperfections (including participation costs, transaction costs, asymmetric information, funding constraints, and search frictions, all of which affect LPs) impact quantitative measures of liquidity such as price impact (λ) and price reversal (γ).[46]

Endogenous Liquidity Provision:
In this paradigm, liquidity is not primarily supplied by dedicated intermediaries but arises organically from the actions of regular market participants or is an inherent feature of the market mechanism itself.

- **Mechanisms:**
  - *Pure Limit Order Books (LOBs):* Liquidity is constituted by the collection of resting limit orders placed by traders who are willing to trade at specific prices but are not necessarily professional market makers.[23] The depth and tightness of the market depend entirely on the collective willingness of participants to post these passive orders. A dynamic cycle can emerge where the presence of limit orders attracts market orders (consuming liquidity), and the resulting trades potentially attract new limit orders (replenishing liquidity).[24]
  - *Automated Market Makers (AMMs):* Liquidity is embedded within the mechanism's design, specifically in the pool of assets governed by a predefined mathematical rule (the invariant function).[3] While external LPs

contribute the assets to the pool, the AMM algorithm itself provides the continuous price quotations and the standing offer to trade against the pool.[5] Examples include CFMMs (like constant product) and LMSR-based AMMs.[5]

- **Theoretical Analysis (Net Impact):** Endogenous liquidity mechanisms offer potential advantages. They can eliminate the explicit bid-ask spread charged by dealers, although implicit costs like AMM trading fees [16] or price impact in LOBs still exist. AMMs, in particular, democratize liquidity provision, allowing anyone with the required assets to become an LP and earn fees.[73] They also guarantee continuous price quoting and the ability to trade at any time (though potentially with high slippage).[5] If liquidity provision is distributed among many participants (as in a deep LOB or a widely-held AMM pool), the market might be more resilient to the withdrawal of any single provider compared to a market reliant on a few dominant MMs.[24]
  However, endogenous liquidity also has drawbacks. Liquidity in pure LOBs can be thin, volatile, and prone to gaps if participation is low or if participants are unwilling to post limit orders, especially during stress.[24] AMMs introduce unique costs and risks: traders face *slippage*, where the execution price moves against them, particularly for large trades [74], and LPs face *impermanent loss* risk.[5] While AMMs provide continuous quotes, the effective liquidity (amount tradable with low slippage) can vary greatly depending on pool size and the AMM's specific curve.[5] Price discovery in AMMs might lag behind markets with sophisticated HFT MMs, although arbitrageurs help align AMM prices with external venues.[5] Furthermore, theoretical models suggest that purely endogenous liquidity provision in the presence of frictions like asymmetric information or limited enforcement can be inherently inefficient, leading to insufficient liquidity to support optimal investment and economic activity.[79] Decentralized OTC markets relying on search mechanisms for endogenous liquidity provision also suffer from inefficiencies due to search frictions and externalities.[46]
- **Mathematical Exploration:** LOB dynamics are often modeled using tools from queueing theory, analyzing order arrival rates, cancellation rates, and execution probabilities.[24] AMM analysis focuses heavily on the properties of the invariant function (e.g., its curvature determining slippage and IL [5]), deriving formulas for slippage and IL, and modeling optimal strategies for LPs and arbitrageurs.[5] Models exploring endogenous liquidity often incorporate specific frictions like adverse selection or limited contract enforcement to explain why liquidity might be suboptimal.[79]

Interaction of Fee Mechanisms:
Fee structures are critical in both models. For external LPs, the bid-ask spread is the primary

compensation mechanism, covering costs and risks.26 The magnitude of the spread is influenced by factors like volatility, information asymmetry, inventory costs, and competition.2 Wider spreads increase transaction costs and reduce market efficiency.1 In AMMs, explicit trading fees, usually a percentage of the trade value, are charged.16 These fees serve as the primary incentive for LPs to deposit capital and bear the risk of impermanent loss.5 The level of these fees creates a direct trade-off: higher fees increase LP returns (potentially attracting more liquidity) but also increase costs for traders (potentially deterring volume). Finding the optimal fee structure is a key aspect of AMM design.
It is useful to view external and endogenous liquidity not as a strict dichotomy but as points on a spectrum. Pure LOBs rely on passive endogenous liquidity but often attract active HFT LPs who behave more like external providers.[23] AMMs depend fundamentally on external LPs depositing assets [5], even though the quoting mechanism itself is endogenous to the protocol. The crucial distinction often lies in whether liquidity provision is the provider's core business objective (external) or an ancillary activity or algorithmic function (endogenous).

Both approaches involve significant externalities. External LPs generate positive externalities through tighter spreads and depth [72], benefiting all traders, but they extract value through the spread [26] and can impose negative externalities through fragility risk.[48] Endogenous systems like AMMs allow broader participation in liquidity provision [73] but create specific negative externalities, such as the divergence loss imposed by arbitrageurs on passive LPs.[5] The presence of these externalities suggests that market outcomes, whether driven by external or endogenous liquidity, may not be inherently socially optimal. Frictions in search markets lead to inefficient private provision [81], and firms issuing assets may not internalize the impact on secondary market liquidity.[81] This points towards the potential value of careful mechanism design or regulation to align private incentives with broader market efficiency and stability goals.[11]

For the design of a blockchain-based distribution market, the choice of liquidity provision model is paramount. The complexity involved in pricing and hedging probability distributions might favor sophisticated external LPs.[26] However, the DeFi ethos often prioritizes decentralized, permissionless, endogenous liquidity provision via AMMs.[3] The market designer must carefully weigh the potential benefits of expert pricing and tighter spreads from professional LPs against the decentralization, accessibility, and potentially different risk profile offered by an AMM-based endogenous model. A critical task will be designing a mechanism that adequately incentivizes liquidity provision, whether external or endogenous, given the unique risks associated with trading probability distributions, linking back to optimal AMM design

principles.[5]

V. Parimutuel Markets: A Detailed Examination

Rigorous Definition:
The parimutuel market structure is a distinct form of wagering or trading mechanism characterized by the pooling of all stakes placed on the various possible outcomes of a future event.4 Originating in 19th century France 29, it operates on the principle of "betting amongst ourselves." After the collection of all wagers, a predetermined commission or fee (the "house take" or "track take") is typically deducted by the market organizer.4 The remaining net pool is then divided proportionally among those participants who placed wagers on the actual winning outcome.4 A defining characteristic is that the final payout odds for any given outcome are not fixed in advance but are determined endogenously only after all betting has ceased, based on the relative amounts wagered on each outcome compared to the total net pool.39 In operational terms, parimutuel markets are often implemented as call auctions, where betting occurs over a defined period, odds fluctuate based on incoming wagers, and the final prices (odds) are set at the close of betting.29
Operational Prerequisites:
The functioning of a parimutuel market requires several key components:

1. A well-defined event with a set of mutually exclusive and exhaustive possible outcomes.
2. A mechanism to accept and record wagers (stakes) placed on each specific outcome.
3. A system for aggregating all stakes into a central pool (or separate pools for different bet types).[85]
4. A clear rule for deducting the organizer's commission or take-out rate from the gross pool.
5. A formula for calculating the final payout odds for the winning outcome *j*: Typically, Odds_j = (Total Pool - House Take) / (Total Amount Wagered on Outcome *j*).[4] Note that odds are often displayed as (Odds_j - 1) to 1.[4]
6. A reliable process for determining the actual outcome of the event.
7. A mechanism for distributing the net pool proportionally to the holders of winning wagers. Often, participation involves purchasing standardized tickets or contracts, frequently at a fixed initial price (e.g., $1 or 1 franc per ticket), with the supply of these initial tickets being effectively unlimited.[4]

Theoretical Properties:

- **Payout Structure:** The payout for a winning bet is inversely proportional to the amount wagered on that specific outcome relative to the total pool. Heavy betting on one outcome drives its odds down, while lightly backed outcomes offer potentially high payouts if they win.[4] This contrasts sharply with *fixed-odds*

markets, where a bookmaker sets the odds before the bet is placed, and these odds remain constant for that specific wager regardless of subsequent betting patterns.[39]

- **Risk Management:**
  - *Organizer:* The parimutuel structure inherently minimizes financial risk for the market organizer.[29] Since payouts are funded entirely by the pool of wagers (less the house take), the organizer is not exposed to losses resulting from the event's outcome. This contrasts significantly with traditional bookmakers, who bear the risk of paying out winning bets from their own capital [82], and also with certain AMM designs (like LMSR) where the market maker subsidizes liquidity and faces bounded potential losses.[9]
  - *Participants:* Bettors face *price uncertainty*, as the final odds are unknown until betting concludes.[82] Their potential payout is affected by the collective actions of all other bettors. The risk associated with an outcome is effectively shared among all participants in the pool.
- **Liquidity:** Parimutuel markets typically exhibit *infinite buy-in liquidity* at the initial ticket price.[4] Any participant can place a wager on any outcome at any time during the betting period without needing a specific counterparty to take the other side.[35] However, a major limitation is the general lack of *exit liquidity*. Standard parimutuel systems do not allow participants to sell or cash out their bets before the event is resolved.[35] This differs from order book markets where limit orders provide exit opportunities, and AMMs which allow selling back to the pool.[73] Innovations like Dynamic Parimutuel Markets (DPMs) have been proposed specifically to address this lack of exit liquidity by hybridizing parimutuel features with continuous trading capabilities.[35]
- **Price Discovery:** The odds in a parimutuel market, which can be converted into implied probabilities (e.g., Implied Probability_j = (Amount Bet on Outcome *j*) / (Total Pool - House Take)), reflect the aggregate "weight of money" or collective opinion of the betting public.[4] Information aggregation occurs as informed participants, believing an outcome is underpriced (odds are too high), place larger or more numerous bets, thereby driving down the odds (and raising the implied probability) for that outcome.[4] However, this price discovery process has limitations: it is primarily *retrospective*, as final odds are only fixed at the market's close [82], and it can be subject to systematic biases, such as the favorite-longshot bias.[86] The lack of continuous price adjustment and exit opportunities may make the information aggregation less dynamic than in continuous markets.[35]

Comparison with Other Structures:

- **vs. Order Book Markets:** Parimutuel markets typically use a call auction format,

contrasting with the continuous trading of LOBs.[29] Price determination relies on proportional pool shares, not the matching of individual buy and sell orders. There is no explicit bid-ask spread. Organizer risk is negligible in parimutuel, unlike the operational risks and participant risks in LOBs. While entry liquidity is high in parimutuel, exit liquidity is generally absent, whereas LOBs allow selling via limit or market orders.

- **vs. AMM-based Markets:** Both can offer high liquidity for initiating trades. However, AMMs provide continuous price quotes based on their internal state (inventory levels and the invariant function) [5], while parimutuel odds evolve based on cumulative betting and finalize only at the close.[82] AMMs permit traders to exit positions by selling back to the pool [73], a feature absent in standard parimutuel markets.[35] Risk profiles differ significantly: AMM LPs face impermanent loss [5], while the parimutuel organizer faces minimal risk.[30] Some AMMs, particularly LMSR, share conceptual links with parimutuel ideas, evolving from scoring rules or being described as dynamic parimutuel mechanisms.[9] DPMs represent a deliberate attempt to merge the parimutuel risk model with AMM/CDA dynamism.[35]
- **vs. Fixed-Odds (Bookmaker) Markets:** The core difference lies in price setting. Parimutuel odds are *endogenous*, determined by the collective bets.[39] Fixed-odds are *exogenous*, set by the bookmaker before the bet.[86] Consequently, the risk burden falls on the bookmaker in fixed-odds markets [82], whereas it is diffused among bettors in parimutuel markets, with the organizer being shielded.[30] Price discovery also differs: parimutuel odds directly reflect aggregate betting patterns [4], while fixed-odds reflect the bookmaker's assessment of probabilities, adjusted for desired profit margin and anticipated adverse selection from informed bettors.[39] Empirical studies comparing the two have found differing payout characteristics, with some suggesting parimutuel markets may offer better returns on longshots under certain conditions.[90]
- **vs. Prediction Markets (General):** Parimutuel is *one specific mechanism* that can be used to implement prediction or betting markets.[4] Other common mechanisms include Continuous Double Auctions (CDAs) [36] and various types of AMMs, most notably LMSR in the prediction market context.[37]

Advantages and Disadvantages:
The primary advantage of the parimutuel structure is the mitigation of risk for the market organizer.30 Its operational simplicity for participants (buying tickets/shares) 4 and the potential for high payouts on correctly chosen, unpopular outcomes are also benefits. Key disadvantages include the lack of price certainty for bettors until the pool closes 82, the inability to exit positions before event resolution in standard implementations 35, potentially less dynamic or continuous price discovery compared to other mechanisms 35, and

susceptibility to the favorite-longshot bias.39
Favorite-Longshot Bias:
This well-documented empirical anomaly in betting markets refers to the tendency for low-probability outcomes (longshots) to attract a disproportionately high amount of betting volume relative to their objective chances of winning, resulting in odds that are "too short" (lower payout than actuarially fair) and thus lower average returns for longshot bettors. Conversely, high-probability outcomes (favorites) tend to be under-bet, offering odds that are "too long" (higher payout than actuarially fair) and thus higher average returns.39 Proposed explanations include bettors exhibiting risk-loving preferences for small bets with large potential payouts 86, cognitive biases, or information-based theories where informed bettors ("insiders") face constraints (e.g., wealth, betting limits) that prevent them from fully correcting the mispricing, particularly on favorites, or where adverse selection problems are more severe for bookmakers on longshots.39 The specific mechanism driving the bias may differ between parimutuel and fixed-odds structures.39
A key insight emerging from this analysis is the inherent trade-off in the parimutuel design between organizer risk and market dynamism. The structure excels at transferring risk away from the organizer and onto the participants.[30] However, this is achieved at the cost of features common in continuous markets, namely price certainty during trading, dynamic price adjustment reflecting information instantaneously, and the ability for participants to manage their positions by exiting before final resolution.[35] This highlights a fundamental design choice: prioritizing organizer solvency versus participant flexibility and continuous price discovery.

For blockchain applications, the parimutuel model's risk-neutrality for the organizer is appealing, as it aligns with the goal of creating autonomous, self-sustaining systems without requiring a heavily capitalized, risk-bearing entity. Smart contracts can readily implement the pooling and payout logic.[4] However, the standard parimutuel model's lack of exit liquidity [35] clashes with the expectations of users in the DeFi space, who are accustomed to the continuous trading and position management offered by AMMs.[73] This tension likely explains the limited adoption of pure parimutuel systems in DeFi and the interest in hybrid models like DPMs [35] or the use of AMMs like LMSR that offer related properties (like bounded loss for the MM) within a more dynamic trading framework.[30]

VI. Continuous Outcome vs. Distribution Markets

Defining the Concepts:
The distinction between markets dealing with continuous outcomes and the specific concept of "distribution markets" is crucial for understanding the evolving landscape of prediction and information markets.

- **Continuous Outcome Market:** This is a general category for any market where

the event being predicted or traded upon can result in an outcome that falls anywhere along a continuous numerical scale, rather than being confined to a discrete set of possibilities.[91] Examples of continuous outcomes include the future price of an asset, the temperature at a specific time, the percentage of votes a candidate will receive, or the magnitude of an economic indicator like inflation or GDP growth. This contrasts fundamentally with traditional binary ("Yes/No") or multinomial ("Candidate A/B/C wins") prediction markets.[36]
- **Distribution Market (Paradigm Proposal):** As specifically proposed by Paradigm [6], a distribution market is a *subtype* of continuous outcome market. Its defining characteristic is that participants trade financial instruments whose value is explicitly tied to the *entire probability distribution* over the continuous range of possible outcomes.[6] Instead of betting on whether the outcome will fall above or below a certain value, or within a specific range, traders in a distribution market express and trade their beliefs about the likelihood of *every* possible outcome simultaneously, represented by a probability density function or a related mathematical function.[6] The traded asset itself embodies a probabilistic forecast across the full spectrum.

Mechanisms for Trading Continuous Outcomes (Non-Distribution):
Several established mechanisms allow participants to trade based on continuous outcomes without explicitly trading the full probability distribution:
- **Discretization:** The continuous range of outcomes is partitioned into a finite number of intervals or "bins." Standard prediction market contracts (e.g., binary options paying $1 if the outcome falls in a specific bin) are then issued for each bin.[93] While simple to implement using existing market structures, this approach forces a potentially arbitrary discretization, limiting the expressiveness of traders whose beliefs might fall near bin boundaries or who wish to express finer-grained predictions.[93] The choice of bin size involves a trade-off between granularity and market complexity/liquidity per bin.
- **Index Contracts:** These contracts have a payout that scales linearly (or according to some other predefined function) with the realized numerical value of the outcome.[91] For example, a contract might pay $0.01 for every point the S&P 500 index closes at on a future date. The equilibrium price of such a contract in an efficient market reflects the market's consensus expectation of the *mean* (average) value of the outcome.[91] By trading multiple index contracts with different payout functions (e.g., one paying based on the outcome y, another based on y^2), it's theoretically possible to infer higher-order moments of the market's belief distribution, such as the variance.[94]
- **Spread Betting:** In this format, a "spread" or cutoff value is proposed (often by

the market maker). Participants bet on whether the final outcome will be higher or lower than this cutoff point.[91] The payout typically depends on how far the outcome deviates from the cutoff. The equilibrium price or cutoff point in a spread betting market tends to reflect the market's expectation of the *median* value of the outcome.[91]

- **Range Bets / Binary Options on Ranges:** These are simple binary contracts that pay a fixed amount (e.g., $1) if the final outcome falls within a specified range (e.g., inflation between 2% and 3%) and zero otherwise.[94] A collection of such contracts covering different ranges can be used to approximate the market's belief about the probability distribution, but requires creating and maintaining liquidity for potentially many individual contracts.[94]

Mechanism for Trading Distributions (Paradigm Proposal):
Paradigm's proposal outlines a novel mechanism specifically designed for trading full probability distributions over continuous outcomes 6:

- **Core Concept:** Participants trade abstract "outcome function tokens," where a trader's position is represented by a function mapping each possible outcome value to the quantity of tokens held for that outcome.[6] This function effectively represents the trader's contribution to the market's overall belief distribution.
- **AMM Approach:** The market operates using a Constant Function Market Maker (CFMM) that works directly with these functions.[6]
- **Invariant Function:** The proposed invariant is based on the L2 norm of the aggregate position function held by the AMM. That is, the integral of the square of the aggregate position function across all possible outcomes ($\int f(x)^2dx$) is kept constant (analogous to $x^2 + y^2 = k^2$ in two dimensions).[6]
- **Trading Process:** When a trader wishes to express a belief different from the current market consensus (represented by the AMM's aggregate function), they execute a trade that adds their desired distribution shape to the AMM's function, while ensuring the L2 norm invariant is maintained. The cost of this trade is determined by the AMM's pricing rule derived from the invariant. The trader profits or loses based on where the actual outcome eventually falls relative to the distribution they effectively "bought" or "sold".[6]
- **Pricing:** The market's consensus probability distribution is implicitly encoded in the shape of the aggregate position function maintained by the AMM. The marginal cost of acquiring tokens for a specific outcome reflects the market's current implied probability density at that point.
- **Collateralization:** Due to the potentially infinite range of outcomes and the nature of the AMM, traders are required to post collateral to cover potential losses and ensure the market remains solvent.[6]

Comparison: Characteristics, Benefits, Drawbacks:

Feature	Non-Distribution Continuous Markets (Index/Spread/Range/Discretized)	Distribution Markets (Paradigm Proposal)
Expressiveness	Limited: Captures mean, median, or probabilities for predefined bins/ranges.[91]	High: Allows trading on the entire probability distribution shape.[6]
Information	Reveals central tendency (mean/median) or binned probabilities.[91] Higher moments inferable with multiple contracts.[94]	Aims to reveal the full consensus probability distribution, including variance, skewness, etc..[6]
Complexity	Relatively simpler mechanisms and contracts.[91]	Theoretically complex; involves functional analysis, novel AMMs.[6]
Liquidity/MM	Can use standard LOBs or simpler AMMs. Discretization can fragment liquidity.[93]	Requires specialized AMMs (e.g., L2-norm AMM). Liquidity provision for functions is challenging.[6]
Practicality	Existing implementations (e.g., Kalshi, spread betting platforms).[91] Discretization widely used.[93]	Largely theoretical/nascent. Faces challenges highlighted by impossibility results for continuous MMs.[6]
Benefits	Simpler to implement and understand. Can provide useful point estimates (mean/median).	Potentially richer information aggregation. More nuanced risk expression and hedging.
Drawbacks	Limited information capture. Discretization is arbitrary and potentially inaccurate.[93]	High complexity. Potential for unbounded MM loss without careful design/collateralization.[6]

		Practical feasibility unproven.

Trading Continuous Outcomes without Full Distributions:
Yes, markets can and do facilitate betting on continuous outcomes without requiring participants to trade full probability distributions. As detailed above, index contracts, spread betting, range binaries, and discretization are all established methods that allow participants to express views and take positions based on continuous variables, focusing on specific parameters (mean, median) or ranges rather than the entire distribution function.91

The development of distribution markets [6] represents a significant conceptual leap, aiming for the ideal of capturing the market's complete state of belief about a continuous variable. While simpler mechanisms like index or spread contracts provide valuable, albeit limited, information about central tendencies [91], a full distribution reveals much more about perceived uncertainty, including the likelihood of tail events, variance, skewness, and potential multimodality.[92] This richer information set holds substantial potential value for sophisticated forecasting, risk management, and decision-making under uncertainty.

However, the transition from finite or discrete state spaces to continuous ones introduces fundamental challenges.[93] Handling an infinite number of potential outcomes complicates mechanism design, particularly for automated market makers. Standard AMM invariants like x*y=k are not directly applicable to functions representing distributions.[6] Ensuring market maker solvency becomes more difficult, as highlighted by theoretical impossibility results showing that it's impossible to simultaneously satisfy all desirable properties—such as responsiveness to trades, bounded loss for the market maker, freedom from arbitrage, and the ability to bet on arbitrary intervals—in a fully general continuous-outcome market maker with binary payoffs.[93] Paradigm's L2-norm AMM [6] is an innovative attempt to navigate these challenges, likely relying on specific mathematical properties of the L2 norm and potentially requiring restrictions (e.g., focusing on specific families of distributions like Normal or Lognormal [6]) and robust collateralization mechanisms [6] to ensure tractability and solvency.

From a terminological standpoint, it is essential to distinguish the general category of "continuous outcome markets" from the specific, more ambitious concept of "distribution markets" as defined by Paradigm.[6] While the latter is a subset of the former, it represents a distinct approach where the traded instrument is the probability distribution itself, rather than a simpler claim derived from the continuous outcome.

VII. Comparative Analysis of AMM Mechanisms: Constant Product vs. LMSR

Automated Market Makers (AMMs): Overview
Automated Market Makers represent a paradigm shift in market design, particularly within the DeFi ecosystem. Instead of relying on traditional limit order books or human intermediaries, AMMs employ algorithmic protocols, typically encoded in smart contracts, to provide liquidity and determine asset prices based on predefined mathematical relationships.3 The core components of an AMM are:

1. **Liquidity Pools:** Reserves of two or more assets held by the smart contract, contributed by users known as Liquidity Providers (LPs).[5]
2. **Trading/Invariant Function:** A mathematical rule that defines the relationship between the quantities of assets in the pool and governs how prices are set. Trades are executed against the pool, altering the reserves, but must maintain the condition defined by this function (or invariant).[3]
3. **Liquidity Providers (LPs):** Users who deposit assets into the pools, enabling trades to occur. In return for providing capital and bearing associated risks, LPs typically earn a share of the trading fees generated by the pool.[5]

AMMs have gained prominence in DeFi due to their advantages in the blockchain context: they are computationally less intensive than managing a full LOB, they can guarantee liquidity (at some price) even for thinly traded assets, and they enable permissionless participation in liquidity provision.[3]

Constant Function Market Makers (CFMMs):
CFMMs constitute a broad class of AMMs defined by the presence of a trading function, f, evaluated on the vector of reserves, R. A trade, represented by vectors of tendered assets (Δ) and received assets (Λ), is considered valid only if the trading function's value remains constant (or changes predictably based on fees), i.e., $f(R - \Lambda + \Delta) = f(R)$.10

- **Constant Product Market Maker (CPMM - e.g., Uniswap V2):** This is arguably the most well-known and foundational CFMM design.[7]
  - *Mechanism:* For a pool with two assets, X and Y, the CPMM maintains the invariant $x * y = k$, where x and y are the quantities of reserves and k is a constant determined by the initial liquidity provision.[10] The instantaneous price of asset Y in terms of asset X is given by the ratio of reserves, $Px/Py = y/x$ (or marginal rate of substitution $|dy/dx| = y/x$), ignoring fees.[5]
  - *Key Properties:*
    - *Liquidity Concentration:* The CPMM provides liquidity across the entire price range from zero to infinity.[98] While ensuring assets are always available for trade, this spreads liquidity thinly and is capital-inefficient, especially for asset pairs with stable relative prices (like stablecoin pairs).[73] Designs like Uniswap V3 introduced *concentrated liquidity*, allowing LPs to

specify price ranges for their capital, improving efficiency.[7]
  - *Slippage:* The price impact experienced by a trader increases with the size of the trade relative to the total liquidity (value of *k*) in the pool.[74] Slippage is directly related to the curvature of the constant product curve; larger trades move the price further along the curve.[5]
  - *Impermanent Loss (IL) / Divergence Loss:* This represents the potential loss LPs incur compared to simply holding their initial assets outside the pool when the relative market price of the pooled assets changes.[5] It arises because arbitrageurs trade with the pool to align its internal price with external market prices, effectively buying the asset that has become relatively cheaper in the pool (more expensive outside) and selling the one that has become relatively more expensive in the pool (cheaper outside). This rebalancing process systematically extracts value from LPs when prices diverge.[5] The magnitude of IL depends on the degree of price change and the specific CFMM curve.[5]
  - *Price Discovery:* CPMM prices react to incoming trades, reflecting net buying or selling pressure. Arbitrageurs play a vital role by ensuring that the CPMM price tracks prices on external, potentially more liquid, exchanges, thus incorporating broader market information.[5]
- *LP Role & Incentives:* LPs deposit assets into the pool, typically in proportion to the current reserve ratio (reflecting the current price).[74] They receive LP tokens representing their proportional claim on the pool's assets and future fees.[16] The primary incentive is earning trading fees, which must compensate for the risk of impermanent loss.[5] The initial and ongoing capital in the pool is provided entirely by these external LPs.[5]
- *Axiomatic View:* The class of CFMMs including CPMMs is characterized by the axioms of *independence* (terms of trade for a subset of assets don't depend on reserves of other assets) and *scale invariance* (exchange rates are invariant to proportional scaling of all reserves).[5] CPMMs fall within the broader category of Constant Elasticity of Substitution (CES) functions or Constant Elasticity of Variance Market Makers (CEMMs).[5] The CPMM has a constant inventory elasticity of 1.[10]

Logarithmic Market Scoring Rule (LMSR) AMMs:
LMSR-based AMMs are predominantly used in prediction markets to aggregate information and forecast probabilities.3 They are derived from the theory of proper scoring rules, which incentivize truthful reporting of beliefs.8

- *Mechanism:* Instead of reserves, the LMSR market maker maintains an internal state representing the quantity of contracts purchased for each possible

outcome of an event, denoted by the vector *q*. A *cost function*, *C(q)*, determines the total cost traders have paid to reach the current state *q*. The price p_i for a contract on outcome *i* is the marginal cost of purchasing that contract: $p_i = \partial C(q)/\partial q_i$.[9] For the LMSR, the cost function is $C(q) = b * \log(\Sigma_j \exp(q_j / b))$, where $b > 0$ is the crucial *liquidity parameter*.[9] The resulting price for outcome *i* is $p_i = \exp(q_i / b) / \Sigma_j \exp(q_j / b)$. These prices naturally sum to 1, representing a probability distribution over the outcomes.[9]

- *Key Properties:*
  - *Liquidity Concentration:* The sensitivity of prices to trades (i.e., liquidity) is controlled by the parameter *b*.[9] A larger *b* value implies lower price sensitivity (less slippage) for a given trade size, meaning the market maker provides more liquidity but also potentially bears more risk.[9] LMSR tends to concentrate liquidity around the current probability estimate, unlike the uniform spread of CPMM.[98]
  - *Slippage:* Price slippage is inversely related to the liquidity parameter *b*. Smaller *b* values lead to greater price impact for trades.[67]
  - *Market Maker Risk/Loss:* The entity operating the LMSR (the "market maker" or organizer) faces a potential net loss. This loss is bounded and, in the worst case, cannot exceed $b \log(N)$, where N is the number of distinct outcomes.[9] This maximum loss occurs if the market starts at a uniform probability distribution and the true outcome was actually certain. This bounded loss represents the cost or subsidy the organizer pays to incentivize information aggregation.[33] This differs fundamentally from the impermanent loss faced by CFMM LPs, which depends on price volatility and is borne by the LPs themselves.[73]
  - *Price Discovery:* LMSR prices are designed to represent probabilities.[67] The underlying scoring rule mechanism provides incentives for myopic, risk-neutral traders to trade such that the market prices reflect their true beliefs.[9] As traders interact with the market based on new information or differing beliefs, the prices (probabilities) update dynamically.[37] LMSR markets are often considered highly effective at information aggregation, particularly in environments with relatively few traders (thin markets).[9]
- *LP Role & Incentives:* In the standard LMSR model, there are no "liquidity providers" in the same sense as in CFMMs.[75] The market maker is the mechanism itself, and its ability to absorb trades (its liquidity) is determined by the parameter *b*, which is set by the market organizer.[9] The capital required to potentially cover the maximum loss ($b \log N$) must be provided or guaranteed by the organizer; this acts as a form of initial subsidy or backing for the market's liquidity.[9] Participants are traders who buy and sell contracts, not LPs contributing assets to a pool.

- *Axiomatic View:* LMSR is characterized by the axioms of *independence* and *translation invariance*.[10] Translation invariance requires that the cost of buying a portfolio that pays $1 regardless of the outcome (i.e., buying one share of every contract) is constant, reflecting the use of an external numéraire (like cash).[10]

Comparative Analysis:

The fundamental difference lies in their operational basis and typical application. CFMMs manage pools of *assets* using *trading functions* to maintain an invariant, primarily for facilitating swaps between those assets in DeFi.[7] LMSR manages the state of *outstanding contracts* using a *cost function* derived from a scoring rule, primarily for eliciting and aggregating *probabilities* in prediction markets.[9]

This leads to different models for liquidity and risk. CFMM liquidity is provided by LPs depositing assets, who bear the risk of impermanent loss due to relative price changes.[5] LMSR liquidity is provided by the market maker mechanism itself, parameterized by *b*, with the market maker bearing a bounded potential loss as a subsidy for information.[9] Consequently, the role of external LPs is central to CFMMs for providing capital [5], while in LMSR, the organizer provides the initial backing represented by *b*.[9]

Prices also have different interpretations: relative exchange rates in CFMMs [5] versus probabilities in LMSR.[67] Axiomatically, this difference is captured by scale invariance (suitable for relative value in asset pools) versus translation invariance (suitable for absolute probabilities relative to an external numéraire).[10]

Table: Comparative Overview of AMM Mechanisms (CPMM vs. LMSR)

Feature	**CFMM (Constant Product Example)**	**LMSR (Logarithmic Market Scoring Rule)**
Primary Use Case	Decentralized token swaps (DeFi) [7]	Prediction markets, Information aggregation [3]
Invariant/Basis	Trading function on reserves (e.g., *x*y* = k*) [16]	Cost function on outstanding contracts (*C(q)* based on log score) [9]
Price Formula	Determined by reserve ratio	Marginal cost (∂*C*/∂*q_i*),

	(e.g., *y/x*) [5]	represents probability [9]
Liquidity Source	Assets deposited by LPs into pool [5]	Market Maker mechanism parameterized by 'b' [9]
LP/MM Role	Provide asset pairs to pool, earn fees [70]	Organizer sets 'b', potentially subsidizes market [9]
LP/MM Risk	Impermanent Loss (IL) / Divergence Loss [5]	Bounded Loss (Max: b log N) [9]
Slippage Determinants	Trade size relative to pool depth (k), curve shape [74]	Trade size relative to liquidity parameter (b) [67]
Price Interpretation	Relative exchange rate [5]	Probability of outcome [67]
Key Axioms	Independence, Scale Invariance [5]	Independence, Translation Invariance [10]
Strengths	Simple, permissionless LPing, composable in DeFi [3]	Incentivizes truthful reporting, good for thin markets, bounded MM loss [9]
Weaknesses	Impermanent loss, capital inefficiency (basic CPMM) [74]	Requires MM subsidy/capital, less natural for asset swaps [9]

The axiomatic characterizations provide a deeper understanding, revealing that both CPMM and LMSR can be derived from the common axiom of *independence*, which dictates that trading a subset of assets should only depend on the reserves of those assets.[5] Their divergence stems from the second key invariance property. *Scale invariance*, natural for asset swaps where value is relative within the pool and LP shares act as a numéraire, leads to CFMMs like the CPMM. *Translation invariance*, natural for prediction markets where probabilities are measured against an external numéraire (like currency) and a risk-free portfolio should have a constant value (or cost), leads to LMSR.[10] This highlights how the mathematical structure reflects fundamental economic assumptions about the market's purpose, the nature of value, and the role of liquidity provision, offering crucial insights for designing new market mechanisms.

VIII. Historical Review: Distribution Markets and Complex Claims

Evolution of Prediction Markets:
The concept of using markets to forecast future events has a long history, predating modern financial theory.

- **Early Forms:** Rudimentary forms existed as political betting markets centuries ago. Records indicate betting on papal succession in the 16th century and widespread, often openly conducted, betting on US presidential elections starting as early as the late 19th century, sometimes exceeding stock market volumes.[44] These early markets often used fixed odds set by "betting commissioners" or operated similarly to bookmaker systems.[87]
- **Theoretical Foundations:** The intellectual groundwork for modern prediction markets lies in economic theories emphasizing the role of markets in aggregating dispersed information. Friedrich Hayek's seminal 1945 work, "The Use of Knowledge in Society," argued that prices effectively coordinate economic activity by summarizing vast amounts of decentralized knowledge.[34] This aligns with the Efficient Market Hypothesis (EMH), formally developed later (e.g., Fama 1970 [34]), which posits that market prices rapidly incorporate all available information.[100] Prediction markets leverage this principle, aiming to elicit and aggregate the collective "wisdom of crowds" [36] into market prices that serve as forecasts.[40]
- **Formalization and Early Experiments:** The modern era of prediction markets arguably began with the Iowa Electronic Markets (IEM), established by university researchers in 1988.[40] The IEM provided a platform for real-money trading on political election outcomes and other events, demonstrating empirically that market-derived forecasts often outperformed traditional opinion polls in accuracy.[37]
- **Corporate and Policy Applications:** Following academic interest, corporations began experimenting with internal prediction markets in the 1990s and 2000s (e.g., Hewlett-Packard [19], Eli Lilly [108], Siemens [40]) to forecast project completion dates, sales figures, R&D success, and other internal metrics, often finding them superior to traditional forecasting methods.[40] Policy applications were also explored, most notably the controversial DARPA "Policy Analysis Market" (PAM) intended to predict geopolitical events, which was cancelled due to public outcry over the possibility of betting on terrorism.[15]
- **Mechanism Development:** Early prediction markets often used standard trading mechanisms like Continuous Double Auctions (CDAs).[36] However, thin liquidity often plagued these markets.[37] This spurred the development of automated market makers specifically designed for prediction markets. Robin Hanson's introduction of Market Scoring Rules (MSRs), particularly the Logarithmic Market

Scoring Rule (LMSR), was a key innovation.[8] LMSR provides guaranteed liquidity via an automated market maker while bounding the potential loss for the market organizer.[9] Other mechanisms like Dynamic Parimutuel Markets (DPMs) were also proposed to combine features of parimutuel systems (low organizer risk) with dynamic pricing.[35]

- **Web and Blockchain Era:** The internet enabled the proliferation of public prediction market platforms (e.g., Intrade, Betfair, later PredictIt [62], Kalshi [62]). More recently, blockchain technology has facilitated the creation of decentralized prediction markets (e.g., Augur [3], Gnosis [99], Polymarket [62]), aiming to offer censorship resistance and remove reliance on central operators, though often facing regulatory challenges.[36]

Seminal Works on Contingent Claims Markets:
The theoretical underpinnings for trading on future uncertain events lie in the economics of contingent claims.

- **Arrow-Debreu Model:** The foundational work by Kenneth Arrow, Gérard Debreu, and Lionel McKenzie in the 1950s established the concept of general equilibrium under uncertainty.[14] They introduced the idea of *state-contingent claims*, now often called *Arrow-Debreu (AD) securities*, which are hypothetical assets paying one unit of account if a specific "state of the world" occurs at a future date, and zero otherwise.[14] The model demonstrates that if markets exist for AD securities corresponding to all possible future states (a *complete market*), then any pattern of payoffs across states can be replicated by forming a portfolio of these basic securities.[14] In such a complete market, risk can be efficiently allocated,

Works cited

1. Market microstructure - Wikipedia, accessed on April 12, 2025, https://en.wikipedia.org/wiki/Market_microstructure
2. people.bath.ac.uk, accessed on April 12, 2025, https://people.bath.ac.uk/mnsak/Microstructure.pdf
3. Automated Market Makers in Cryptoeconomic Systems: A Taxonomy and Archetypes - arXiv, accessed on April 12, 2025, https://arxiv.org/html/2309.12818v3
4. Parimutuel Betting Markets as Information Aggregation Devices: Experimental Results, accessed on April 12, 2025, https://www.researchgate.net/publication/24056891_Parimutuel_Betting_Markets_as_Information_Aggregation_Devices_Experimental_Results
5. (PDF) The Economics of Constant Function Market Makers, accessed on April 12, 2025, https://www.researchgate.net/publication/379181185_The_Economics_of_Constant_Function_Market_Makers
6. Distribution Markets - Paradigm, accessed on April 12, 2025,

https://www.paradigm.xyz/2024/12/distribution-markets
7. Constant Function Market Makers: Multi-asset Trades via Convex Optimization, accessed on April 12, 2025, https://www.researchgate.net/publication/369171394_Constant_Function_Market_Makers_Multi-asset_Trades_via_Convex_Optimization
8. Designing Markets for Prediction - AI Magazine, accessed on April 12, 2025, https://ojs.aaai.org/aimagazine/index.php/aimagazine/article/view/2313/2179
9. Logarithmic Market Scoring Rules for Modular Combinatorial Information Aggregation | Request PDF - ResearchGate, accessed on April 12, 2025, https://www.researchgate.net/publication/2837599_Logarithmic_Market_Scoring_Rules_for_Modular_Combinatorial_Information_Aggregation
10. arxiv.org, accessed on April 12, 2025, https://arxiv.org/pdf/2210.00048
11. Optimal Automated Market Makers - University of Warwick, accessed on April 12, 2025, https://warwick.ac.uk/fac/soc/wbs/subjects/finance/phd/ab101/zhengge_zhou_job_market_paper.pdf
12. Market Microstructure: A Review of models - ResearchGate, accessed on April 12, 2025, https://www.researchgate.net/publication/382940748_Market_Microstructure_A_Review_of_models
13. Market microstructure: A survey, accessed on April 12, 2025, https://www.acsu.buffalo.edu/~keechung/MGF743/Readings/Market%20microstructure%20A%20surveyq.pdf
14. The Arbitrage Principle in Financial Economics - American ..., accessed on April 12, 2025, https://pubs.aeaweb.org/doi/10.1257/jep.1.2.55
15. www.princeton.edu, accessed on April 12, 2025, https://www.princeton.edu/~dixitak/Teaching/EconomicsOfUncertainty/Slides&Notes/Slides12.pdf
16. Constant Function Market Makers: Multi-asset Trades via Convex Optimization - Stanford University, accessed on April 12, 2025, https://www-leland.stanford.edu/~boyd/papers/pdf/cfmm.pdf
17. Untitled - IMF eLibrary, accessed on April 12, 2025, https://www.elibrary.imf.org/view/journals/001/1994/060/article-A001-en.pdf
18. robinhanson.com, accessed on April 12, 2025, http://robinhanson.com/biashelp.pdf
19. hanson.gmu.edu, accessed on April 12, 2025, https://hanson.gmu.edu/combobet.pdf
20. hanson.gmu.edu, accessed on April 12, 2025, https://hanson.gmu.edu/insiderbet.pdf
21. Market microstructure: Explained - TIOmarkets, accessed on April 12, 2025, https://tiomarkets.com/en/article/market-microstructure-guide
22. Market Microstructure: A Practitioner's Guide - CiteSeerX, accessed on April 12, 2025, https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=af833345253a6ff75f8f8b94a97e2d694a64a561

23. faculty.haas.berkeley.edu, accessed on April 12, 2025, https://faculty.haas.berkeley.edu/hender/IIROC.pdf
24. MARKET MICROSTRUCTURE OF AN ORDER DRIVEN MARKET - HKU Scholars Hub, accessed on April 12, 2025, https://hub.hku.hk/bitstream/10722/26298/6/FullText.pdf
25. Price Discovery in the Absence of Trading: Insights from the Malta Stock Exchange Pre-opening Period, accessed on April 12, 2025, https://www.bayes.citystgeorges.ac.uk/__data/assets/pdf_file/0017/41543/Johnson.pdf
26. www.acsu.buffalo.edu, accessed on April 12, 2025, https://www.acsu.buffalo.edu/~keechung/MGF743/Readings/Hans%20Stoll,%202003,%20Market%20microstructure.pdf
27. CGFS Publications - The Market Microstructure of Dealership Equity and Government Securities Markets: How They Differ. - Bank for International Settlements, accessed on April 12, 2025, https://www.bis.org/publ/cgfs11gra_b.pdf
28. Market Makers in Financial Markets: Their Role, How They Function, Why They are Important, and the NYSE DMM Difference Steven W., accessed on April 12, 2025, https://www.nyse.com/publicdocs/nyse/NYSE_Paper_on_Market_Making_Sept_2021.pdf
29. Nicholas Economides | Economics of Networks, accessed on April 12, 2025, http://neconomides.stern.nyu.edu/networks/Parimutuel.pdf
30. web.stanford.edu, accessed on April 12, 2025, https://web.stanford.edu/~yyye/scpmfinal.pdf
31. A Parimutuel Market Microstructure for Contingent Claims Trading - NYU, accessed on April 12, 2025, https://web-docs.stern.nyu.edu/salomon/docs/derivatives/S-DRP-01-16.pdf
32. Price discovery - Wikipedia, accessed on April 12, 2025, https://en.wikipedia.org/wiki/Price_discovery
33. Aggregation and Manipulation in Prediction Markets: Effects of Trading Mechanism and Information Distribution - ResearchGate, accessed on April 12, 2025, https://www.researchgate.net/publication/221444757_Aggregation_and_Manipulation_in_Prediction_Markets_Effects_of_Trading_Mechanism_and_Information_Distribution
34. Designing Markets For Prediction - Harvard DASH, accessed on April 12, 2025, https://dash.harvard.edu/bitstreams/7312037c-910d-6bd4-e053-0100007fdf3b/download
35. courses.cs.duke.edu, accessed on April 12, 2025, https://courses.cs.duke.edu/spring07/cps296.3/pennock-ec-2004-dynamic-parimutuel.pdf
36. Prediction Market: Overview, Types, Examples - Investopedia, accessed on April 12, 2025, https://www.investopedia.com/terms/p/prediction-market.asp
37. The Relative Accuracy of LMSR and CDA ... - Harvard DASH, accessed on April 12, 2025, https://dash.harvard.edu/bitstreams/b6ade073-1244-40c1-afb5-9f9ef6740ef0/do

wnload
38. What Is Price Discovery? Definition, Process, and vs. Valuation - Investopedia, accessed on April 12, 2025, https://www.investopedia.com/terms/p/pricediscovery.asp
39. Parimutuel versus Fixed-Odds Markets, accessed on April 12, 2025, https://didattica.unibocconi.it/mypage/upload/48832_20111118_024849_PARIMUTUELVERSUSFIXED-ODDSMARKETS.PDF
40. Prediction markets: An information aggregation perspective to the forecasting problem, accessed on April 12, 2025, https://www.researchgate.net/publication/5173495_Prediction_markets_An_information_aggregation_perspective_to_the_forecasting_problem
41. An Experiment on Prediction Markets in Science - PMC, accessed on April 12, 2025, https://pmc.ncbi.nlm.nih.gov/articles/PMC2794371/
42. Information Aggregation with Costly Information Acquisition, accessed on April 12, 2025, https://www2.aueb.gr/conferences/Crete2024/papers/Galanis.pdf
43. Designing Markets for Prediction - ResearchGate, accessed on April 12, 2025, https://www.researchgate.net/publication/220605668_Designing_Markets_for_Prediction
44. Prediction market - Wikipedia, accessed on April 12, 2025, https://en.wikipedia.org/wiki/Prediction_market
45. October 2015 GFSR Chapter 2: Market Liquidity—Resilient or Fleeting?, accessed on April 12, 2025, https://www.imf.org/External/Pubs/FT/GFSR/2015/02/pdf/c2_v2.pdf
46. www.nber.org, accessed on April 12, 2025, https://www.nber.org/system/files/working_papers/w18251/w18251.pdf
47. Information Jumps, Liquidity Jumps, and Market Efficiency - MDPI, accessed on April 12, 2025, https://www.mdpi.com/1911-8074/15/3/97
48. Shifting tides - market liquidity and market-making in fixed income instruments, accessed on April 12, 2025, https://www.bis.org/publ/qtrpdf/r_qt1503i.htm
49. What Is Scarcity? - Investopedia, accessed on April 12, 2025, https://www.investopedia.com/terms/s/scarcity.asp
50. Scarcity in Economics: Causes & Implications - Tickeron, accessed on April 12, 2025, https://tickeron.com/trading-investing-101/what-is-economic-scarcity-and-how-does-it-shape-our-choices/
51. Law of Supply and Demand in Economics: How It Works - Investopedia, accessed on April 12, 2025, https://www.investopedia.com/terms/l/law-of-supply-demand.asp
52. Consequences of Asset Shortages in Emerging Markets - International Monetary Fund (IMF), accessed on April 12, 2025, https://www.imf.org/external/pubs/ft/wp/2012/wp12102.pdf
53. The Safe Assets Shortage Conundrum - MIT Economics, accessed on April 12, 2025, https://economics.mit.edu/sites/default/files/publications/jep.31.3.29.pdf
54. Scarcity effects of QE: A transaction-level analysis in the Bund market - Bank for International Settlements, accessed on April 12, 2025,

https://www.bis.org/publ/work625.pdf
55. Collateral Easing and Safe Asset Scarcity: How Money Markets Benefit from Low-Quality Collateral - European Central Bank, accessed on April 12, 2025, https://www.ecb.europa.eu/press/conferences/shared/pdf/20241107_money_markets/Greppmair_paper.pdf
56. Safe Asset Scarcity and Monetary Policy Transmission - European Central Bank, accessed on April 12, 2025, https://www.ecb.europa.eu/press/conferences/shared/pdf/20231109_money_markets/Nguyen_paper.en.pdf
57. Scarcity Effects of Quantitative Easing on Market Liquidity: Evidence from the Japanese Government Bond Market in: IMF Working Papers Volume 2018 Issue 096 (2018), accessed on April 12, 2025, https://www.elibrary.imf.org/view/journals/001/2018/096/article-A001-en.xml
58. Scarcity of Assets, Private Information, and the Liquidity Trap - Department of Economics | Ole Miss, accessed on April 12, 2025, https://economics.olemiss.edu/wp-content/uploads/sites/209/2018/05/scarcity-assets-private-2.pdf
59. A Detailed Guide On Synthetic Assets - Rejolut, accessed on April 12, 2025, https://rejolut.com/blog/a-detailed-guide-on-synthetic-assets/
60. Tokenomics |The Ultimate Guide to Crypto Economy Design - Rapid Innovation, accessed on April 12, 2025, https://www.rapidinnovation.io/post/tokenomics-guide-mastering-blockchain-token-economics-2024
61. An Analysis of Upgradeability, Oracles, and Stablecoins in the Ethereum Blockchain - Concordia University, accessed on April 12, 2025, https://users.encs.concordia.ca/~mmannan/student-resources/Thesis-MASc-Salehi-2022.pdf
62. Great Wall of Numbers | Business Opportunities and Challenges in Emerging Markets, accessed on April 12, 2025, https://www.ofnumbers.com/
63. What is Elasticity? - 2020- Robinhood, accessed on April 12, 2025, https://robinhood.com/us/en/learn/articles/46ambQ0ZI0sE6XS4c5VcVS/what-is-elasticity/
64. Elasticity of supply | economics-hl | ib - Sparkl, accessed on April 12, 2025, https://www.sparkl.me/learn/ib/economics-hl/elasticity-of-supply/revision-notes/1481
65. Price elasticity of demand - Wikipedia, accessed on April 12, 2025, https://en.wikipedia.org/wiki/Price_elasticity_of_demand
66. The Supply and Demand Paradigm (An Intro to Chicago Price Theory) - Lones Smith, accessed on April 12, 2025, https://www.lonessmith.com/wp-content/uploads/2021/05/marketexchange2021.pdf
67. Optimizing the liquidity parameter of logarithmic market scoring rules prediction markets | Emerald Insight, accessed on April 12, 2025, https://www.emerald.com/insight/content/doi/10.1108/jm2-06-2017-0066/full/pdf?title=optimizing-the-liquidity-parameter-of-logarithmic-market-scoring-rules-p

rediction-markets
68. Arbitrage in automated market makers - Frontiers, accessed on April 12, 2025, https://www.frontiersin.org/journals/blockchain/articles/10.3389/fbloc.2024.1455070/full
69. An Optimization-Based Framework for Combinatorial Prediction Market Design - CiteSeerX, accessed on April 12, 2025, https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=d66b8b2405f3849c445b0ebea42bd67f3f1ed166
70. Optimal Automated Market Makers - University of Warwick, accessed on April 12, 2025, https://warwick.ac.uk/fac/soc/wbs/subjects/finance/phd/ab101/optimal_automated_market_makers_4.pdf
71. (PDF) A New Era of Automated Market-Makers (AMM) powered by Non-Fungible Tokens- A Review - ResearchGate, accessed on April 12, 2025, https://www.researchgate.net/publication/367553014_A_New_Era_of_Automated_Market-Makers_AMM_powered_by_Non-Fungible_Tokens-_A_Review_Non-Fungible_Tokens_for_Automated_Market-Makers
72. Liquidity Providers vs Market Makers: What you need to know - WL Global Solutions, accessed on April 12, 2025, https://www.wlglobal.solutions/blog/liquidity-provider-vs-market-maker/
73. Automated Market Making (AMM) vs. Traditional Market Making - Orcabay, accessed on April 12, 2025, https://orcabay.io/blog/amm-vs-traditional-market-making/
74. (PDF) Order Book Inspired Automated Market Making - ResearchGate, accessed on April 12, 2025, https://www.researchgate.net/publication/378663160_Order_Book_Inspired_Automated_Market_Making
75. AMM Types & Differentiations Examples [+Use Cases] - Blaize Tech, accessed on April 12, 2025, https://blaize.tech/blog/amm-types-differentiations/
76. Assessing the Impact of High-Frequency Trading on Market Efficiency and Stability, accessed on April 12, 2025, https://www.oxjournal.org/assessing-the-impact-of-high-frequency-trading-on-market-efficiency-and-stability/
77. THE IMPACT OF LIQUIDITY PROVIDERS ON THE BALTIC STOCK EXCHANGE, accessed on April 12, 2025, https://www.nasdaqbaltic.com/files/riga/Studiju_darbi/Bachelor_Thesis_Timofejeva_Grecuhina_final%5B1%5D.pdf
78. How do Brokers and Liquidity Providers Work Together? - Soft-FX, accessed on April 12, 2025, https://www.soft-fx.com/blog/how-do-brokers-and-liquidity-providers-work-together/
79. Endogenous Liquidity and the Business Cycle - University at Albany, accessed on April 12, 2025, https://www.albany.edu/sites/default/files/2019-08/sbigio.pdf
80. Endogenous Liquidity and the Business Cycle - American Economic Association, accessed on April 12, 2025, https://pubs.aeaweb.org/doi/abs/10.1257/aer.20110035

81. Private and Public Liquidity Provision in Over-the-Counter Markets - Federal Reserve Board, accessed on April 12, 2025, https://www.federalreserve.gov/econres/feds/files/2017033pap.pdf
82. Tyler Patla - The Effect of Market Structure on Pricing Efficiency - Sites@Duke Express, accessed on April 12, 2025, https://sites.duke.edu/djepapers/files/2016/10/Patla.pdf
83. Disagreement and market structure in betting markets: Theory and evidence from European soccer - EconStor, accessed on April 12, 2025, https://www.econstor.eu/bitstream/10419/296686/1/WP23_12.pdf
84. Disagreement and Market Structure in Betting Markets: Theory and Evidence from European Soccer - University College Dublin, accessed on April 12, 2025, https://www.ucd.ie/economics/t4media/WP23_12.pdf
85. Anomalies: Parimutuel Betting Markets: Racetracks and Lotteries - American Economic Association, accessed on April 12, 2025, https://pubs.aeaweb.org/doi/pdf/10.1257%2Fjep.2.2.161
86. web2.econ.ku.dk, accessed on April 12, 2025, https://web2.econ.ku.dk/sorensen/papers/pvfom.pdf
87. Do Prediction Markets Add Do Prediction Markets Add Value to the Forecasting Process - Northfield Information Services, accessed on April 12, 2025, https://www.northinfo.com/documents/321.pdf
88. Market Structure and Prices in Online Betting Markets: Theory and Evidence Tadgh Hegarty* Karl Whelan†, accessed on April 12, 2025, https://www.karlwhelan.com/Papers/Monopoly.pdf
89. A dynamic pari-mutuel market for hedging, wagering, and information aggregation - ResearchGate, accessed on April 12, 2025, https://www.researchgate.net/publication/279714212_A_dynamic_pari-mutuel_market_for_hedging_wagering_and_information_aggregation
90. An Empirical Examination of the Parimutuel Sports Lottery Market versus the Bookmaker Market - ResearchGate, accessed on April 12, 2025, https://www.researchgate.net/publication/46568609_An_Empirical_Examination_of_the_Parimutuel_Sports_Lottery_Market_versus_the_Bookmaker_Market
91. A Primer on Prediction Markets - Wharton Initiative on Financial ..., accessed on April 12, 2025, https://wifpr.wharton.upenn.edu/blog/a-primer-on-prediction-markets/
92. Probability Distribution: Definition, Types, and Uses in Investing - Investopedia, accessed on April 12, 2025, https://www.investopedia.com/terms/p/probabilitydistribution.asp
93. cs.uwaterloo.ca, accessed on April 12, 2025, https://cs.uwaterloo.ca/~a23gao/papers/axiomatic-WINE10.pdf
94. Prediction Markets - EPFL Moodle, accessed on April 12, 2025, https://moodle.epfl.ch/mod/resource/view.php?id=1044926&redirect=1
95. A Understanding Incentives in Social Computing - acm sigai, accessed on April 12, 2025, http://sigai.acm.org/static/cnc_abstracts/gao.pdf
96. An Optimization-Based Framework for Automated Market-Making - CMU School of Computer Science, accessed on April 12, 2025,

https://www.cs.cmu.edu/~./sandholm/cs15-892F15/optimization%20framework%20for%20automated%20market%20making.ec11.pdf
97. Optimal Routing for Constant Function Market Makers - Stanford University, accessed on April 12, 2025, https://web.stanford.edu/~boyd/papers/pdf/cfmm-routing.pdf
98. pm-AMM: A Uniform AMM for Prediction Markets - Paradigm, accessed on April 12, 2025, https://www.paradigm.xyz/2024/11/pm-amm
99. Improved Price Oracles: Constant Function Market Makers - Guillermo Angeris, accessed on April 12, 2025, https://angeris.github.io/papers/constant_function_amms_old.pdf
100. Prediction markets: Fundamentals, designs, and applications | Request PDF, accessed on April 12, 2025, https://www.researchgate.net/publication/286352360_Prediction_markets_Fundamentals_designs_and_applications
101. The Efficient Market Hypothesis and Its Critics by Burton G. Malkiel, Princeton University CEPS Working Paper No. 91 April 2003, accessed on April 12, 2025, https://www.princeton.edu/~ceps/workingpapers/91malkiel.pdf
102. PREDICTION MARKETS: Theory, Evidence and Applications. - NTU > IRep, accessed on April 12, 2025, https://irep.ntu.ac.uk/id/eprint/35265/1/12800_Vaughan-Williams.pdf
103. Fundamentals of Prediction Markets | Request PDF - ResearchGate, accessed on April 12, 2025, https://www.researchgate.net/publication/300098615_Fundamentals_of_Prediction_Markets
104. Prediction Markets in Theory and Practice - ResearchGate, accessed on April 12, 2025, https://www.researchgate.net/publication/5136256_Prediction_Markets_in_Theory_and_Practice
105. Morality and Markets: A Comment on Predicting Crime - Journals at the University of Arizona, accessed on April 12, 2025, https://journals.librarypublishing.arizona.edu/arizlrev/article/7256/galley/6687/download/
106. Decision Market Predictions: Separating Efficiency and Bias - CiteSeerX, accessed on April 12, 2025, https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=11ccbef244805fa6fa1fbafa7112dfdba8232c2e
107. Prediction Markets - TIB, accessed on April 12, 2025, https://edocs.tib.eu/files/e01fb10/618682252.pdf
108. Prediction Markets for Corporate Governance - Chicago Unbound, accessed on April 12, 2025, https://chicagounbound.uchicago.edu/cgi/viewcontent.cgi?article=2342&context=law_and_economics
109. Prediction Markets - Kottakkal Farook arts and science college, accessed on April 12, 2025, http://dspace.kottakkalfarookcollege.edu.in:8001/jspui/bitstream/123456789/2879/

1/Prediction%20Markets.pdf
110. Computational Aspects of Prediction Markets - ResearchGate, accessed on April 12, 2025, https://www.researchgate.net/publication/254230659_Computational_Aspects_of_Prediction_Markets
111. An Empirical Study of Dynamic Pari-mutuel Markets: Evidence from the Tech Buzz Game, accessed on April 12, 2025, https://www.researchgate.net/publication/201168851_An_Empirical_Study_of_Dynamic_Pari-mutuel_Markets_Evidence_from_the_Tech_Buzz_Game
112. Prediction Markets: Exploring Insights, Limitations, and Practical Applications – BSIC, accessed on April 12, 2025, https://bsic.it/prediction-markets-exploring-insights-limitations-and-practical-applications/
113. Arrow–Debreu model - Wikipedia, accessed on April 12, 2025, https://en.wikipedia.org/wiki/Arrow%E2%80%93Debreu_model
114. The Arrow-Debreu Model of General Equilibrium - Yale Department of Economics, accessed on April 12, 2025, http://dido.econ.yale.edu/~gean/art/1987-newpalgrave1.pdf
115. "Asset Pricing and Risk Sharing in CompleteAsset Pricing and Risk Sharing in Complete Markets: An Experimental Investigation - Toulouse School of Economics, accessed on April 12, 2025, https://www.tse-fr.eu/sites/default/files/TSE/documents/doc/wp/2017/wp_tse_798.pdf
116. The stylized facts of prediciton markets: analysis of price changes - ePrints Soton - University of Southampton, accessed on April 12, 2025, https://eprints.soton.ac.uk/423710/1/manuscript_REVISION.pdf
117. Multiperiod Securities and the Efficient Allocation of Risk: A Comment on the Black-Scholes Option Pricing Model - National Bureau of Economic Research, accessed on April 12, 2025, https://www.nber.org/system/files/chapters/c4436/c4436.pdf
118. Prediction Markets - ResearchGate, accessed on April 12, 2025, https://www.researchgate.net/publication/4981700_Prediction_Markets
119. Designing Informative Securities - Jennifer Wortman Vaughan, accessed on April 12, 2025, https://www.jennwv.com/papers/informativesecurities.pdf
120. Prediction Markets: Theory and Applications - Harvard DASH, accessed on April 12, 2025, https://dash.harvard.edu/bitstreams/9f4daf73-e061-4262-90fc-54d11b1e11c7/download
121. An Optimization-Based Framework for Combinatorial Prediction Market Design | Yiling Chen, accessed on April 12, 2025, https://yiling.seas.harvard.edu/publications/optimization-based-framework-combinatorial-prediction-market-design
122. X Efficient Market Making via Convex Optimization, and a Connection to Online Learning - Electrical Engineering and Computer Science, accessed on April 12, 2025, https://web.eecs.umich.edu/~jabernet/papers/ACV12.pdf

123. Price formation in field prediction markets: The wisdom in the crowd - University of Cambridge, accessed on April 12, 2025, https://www.repository.cam.ac.uk/bitstreams/6a81facc-f586-4dc0-87a6-53d6d155fb06/download